BIBLIOGRAPHY: MAPPING CLASS GROUP DYNAMICS ON CHARACTER VARIETY

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Generalities on character varieties:

- *"The Symplectic Nature of Fundamental Groups of Surfaces"*, Goldman, 1984 [Gol84] . Construction of the symplectic form on character varieties.
- *"Invariant Functions on Lie Groups and Hamiltonian Flows of Surface Group Representations"*, Goldman, 1986 [Gol86] . Construction of hamiltonian flows associated to invariant functions on character varieties.
- *"Trace Coordinates on Fricke Spaces of Some Simple Hyperbolic Surfaces"*, Goldman, 2009 [Gol09] . Description of trace coordinates for the character variety of some hyperbolic surfaces with values in $SL(2, \mathbb{C})$.
- A Note on Character Varieties, Maret, 2022 [Mar22] . Definitions and properties of character varieties, discussion of the symplectic form.
- *"Character Varieties"*, Sikora, 2012 [Sik12] Discussion of the GIT of character varieties.

Ergodic dynamics of $\mathcal{MCG}$ in the compact case:

- *"Ergodic Theory on Moduli Spaces"*, Goldman, 1997 [Gol97] . Ergodicity of the action of $\mathcal{MCG}$ on the relative character varieties for closed surfaces and target group (product of) $SU(2)$.
- *"Ergodicity of Mapping Class Group Actions on Representation Varieties, I. Closed Surfaces"*, Pickrell and Xia, 2002 [PX02a] . Ergodicity for $\mathcal{MCG}$ on the representation variety for a closed surface and any compact Lie group. Argument of harmonic analysis on compact Lie group.
- *"Ergodicity of Mapping Class Group Actions on Representation Varieties, II. Surfaces with Boundary"*, Pickrell and Xia, 2003 [PX03a] . Ergodicity for $\mathcal{MCG}$ on the relative representation varieties for a surface with boundary and any compact Lie group.
- *"An Ergodic Action of the Outer Automorphism Group of a Free Group"*, Goldman, 2007 [Gol07] . Ergodicity for the action of the outer automorphism of the free group F_n, $n \geq 3$, on the character variety into $SU(2)$.
- *"On Deformations of Fn in Compact Lie Groups"*, Gelander, 2008 [Gel08] . Ergodicity for the action of the outer automorphism of the free group F_n, $n \geq 3$, on the character variety into any compact Lie group.
- *"Ergodicity Of Mapping Class Group Actions On SU(2)-Character Varieties"*, Goldman and Xia, 2011 [GX11] . Another proof of the ergodicity of the action of $\mathcal{MCG}$ on the relative character varieties for closed surfaces and target group (product of) $SU(2)$, which uses the transitivity of the Hamiltonian flows generated by the traces of enough simple closed curves to simplify Goldman's original argument.
- *"Action of the Johnson-Torelli Group on Representation Varieties"*, Goldman and Xia, 2012 [GX12] . Ergodicity of the action of the Johnson subgroup of $\mathcal{MCG}$, the subgroup generated by Denh twists along null-homotopic curves

(in particular for the Torelli group) on the relative character varieties for the 2-holed torus with values in $SU(2)$ and generic peripheric data.

- *"The First Johnson Subgroups Act Ergodically on SU_2-Character Varieties"*, Funar and Marché, 2013 [FM13] . Ergodicity of the action of the Johnson subgroup on the character variety of a closed surface of genus ≥ 2 with values in $SU(2)$.
- On the Torelli Group Action on Compact Character Varieties, Bouilly, 2020 [Bou20] . Ergodicity of the Torelli group (subgroup of $\mathcal{MCG}$ of elements which act trivially on homology) on (a connected component of) the character variety of a closed surface of genus ≥ 2 with values in any (connected, simply connected) compact Lie group.
- *"The Mapping Class Group Action on SU(3)-Character Varieties"*, Goldman, Lawton, and Xia, 2021 [GLX21] . Generalization of the argument of [GX11] for $SU(3)$.
- Non-Ergodicity on SU(2) and SU(3) Character Varieties of the Once-Punctured Torus, Forni et al., 2022 [For+22] . Non-ergodicity of one pseudo-Anosov element on the relative character variety of the one-holed torus with value in $SU(2)$ for the trace paramater small enough. Uses KAM techniques to show the persistence of a neutral fixed point.

Ergodic dynamics of $\mathcal{MCG}$ in the non-compact case:

- *"The Complex-Symplectic Geometry of SL(2,C)-Characters over Surfaces"*, Goldman, 2003 [Gol03b] . Description of the complex case, the complex symplectic structure.
- *"The Modular Group Action on Real* SL *(2)–Characters of a One-Holed Torus"*, Goldman, 2003 [Gol03a] . Precise description of the dynamics of $\mathcal{MCG}$ on the relative character varieties for the one-holed torus and $SL(2, \mathbb{R})$.
- *"The Dynamics of Aut(F_n) on Redundant Representations"*, Gelander and Minsky, 2013 [GM13] . Action of $Out(F_n)$ on the character variety of F_n and study of the redundant representations: they are the representations whose restriction to a proper free factor has a dense image. They show that the action is ergodic for every connected simple k-group, for k a local field. But the action is not weakly mixing for $SL(2, \mathbb{R})$ and $SL(2, \mathbb{C})$. They also show that the action on redundant representations is minimal.

Topological dynamics of $\mathcal{MCG}$:

- *"Topological Dynamics on Moduli Spaces, I"*, Previte and Xia, 2000 [PX00] . For the action of $\mathcal{MCG}$ on the relative character varieties of the one-holed torus, the orbit of a representation with dense image is dense.
- *"Topological Dynamics on Moduli Spaces II"*, Previte and Xia, 2002 [PX02b] . For the action of $\mathcal{MCG}$ on the relative character varieties of a surface (possibly) with boundary and genus $g > 0$, the orbit of a representatio with dense image is dense.
- *"Exceptional Discrete Mapping Class Group Orbits in Moduli Spaces"*, Previte and Xia, 2003 [PX03b] . For some relative character varieties of the four-holed sphere with value in $SL(2, \mathbb{C})$, they prove that there exists a representation with dense image and finite orbit under $\mathcal{MCG}$.
- *"Dynamics of the Mapping Class Group on the Moduli of a Punctured Sphere with Rational Holonomy"*, Previte and Xia, 2005 [PX05] . For the four-holed sphere, for a dense set of peripheric data, for the compact component of the

real points of the associated relative character variety with value in SL(2, $\mathbb{C}$), every orbit of $\mathcal{MCG}$ is dense.

- *"Dynamics of the Mapping Class Group Action on the Variety of PSL(2,C) Characters"*, Souto and Storm, 2006 [SS06] . In the character variety of a closed surface of genus ≥ 2 with value in SL(2, $\mathbb{C}$), the action of $\mathcal{MCG}$ on some nowhere dense topologically perfect subset of the boundary of the quasifuchsian locus is topologically transitive (the closure of the representations whose quotient has no conformally compact end). They also prove that any $\mathcal{MCG}$-invariant meromorphic function on a $\mathcal{MCG}$-invariant open set which contains both convex-cocompact and indiscrete representations is constant.

- *"Transitivity of Normal Subgroups of the Mapping Class Group on Character Varieties"*, Marché and Wolff, 2022 [MW22] . In the character variety of a closed surface of genus ≥ 2 with value in $SU(2)$, they prove that the action of any non-central normal subgroup of $\mathcal{MCG}$ is almost minimal (the orbit of almost every point is dense).

Proper dynamics of $\mathcal{MCG}$:

- *"On Dynamics of Out(Fn) on PSL2(C) Characters"*, Minsky, 2013 [Min13] . Action of $Out(F_n)$ on the character variety of F_n with values in SL(2, $\mathbb{C}$). Introduction of the primitive-stable representations, an open subset strictly containing the Schottky representations, on which $Out(F_n)$ acts properly discontinuously.

- *"Amalgam Anosov Representations"*, Canary, Lee, and Stover, 2017 [CLS17] . Introduction of Amalgam Anosov representations, for Γ a one-ended torsion-free hyperbolic group. They form a domain of discontinuity for the action of $Out(\Gamma)$, containing strictly the Anosov representations.

- Primitive Stable Representations in Higher Rank Semisimple Lie Groups, Kim and Kim, 2020 [KK20] . Primitive stable representations in higher rank.

Complex dynamics:

- *"On the Dynamics of Pseudo-Anosov Homeomorphisms on Representation Varieties of Surface Groups"*, Kapovich, 1998 [Kap98] . In the character variety of a closed surface of genus ≥ 2, a pseudo-Anosov f in $\mathcal{MCG}$ has a hyperbolic fixed point at the representation induced by the hyperbolic structure on the mapping torus of f.

- *"Bers and Hénon, Painlevé and Schrödinger"*, Cantat, 2009 [Can09] . Study of the complex dynamics of the iteration of a pseudo-Anosov on the relative character varieties for the one-holed torus and the four-holed sphere with value in SL(2, $\mathbb{C}$).

- *"Dynamics on Character Varieties and Malgrange Irreducibility of Painlevé VI Equation"*, Cantat and Loray, 2009 [CL09] . Study of the complex geometric structures invariant by $\mathcal{MCG}$ on the relative character varieties for the one-holed torus and the four-holed sphere with values in SL(2, $\mathbb{C}$).

- Dynamics of Groups of Automorphisms of Character Varieties and Fatou/Julia Decomposition for Painlevé 6, Rebelo and Roeder, 2022 [RR22] . Study of the Julia/Fatou set and locally discrete/non-locally discrete dichotomy for the action of $\mathcal{MCG}$ on the relative character varieties for the one-holed torus and the four-holed sphere with value in SL(2, $\mathbb{C}$).

- Automorphisms of Character Varieties, Marché and Simon, 2019 [MS19] The automorphism group of the character variety of a closed surface with values in $SL(2, \mathbb{C})$ is (almost) the mapping class group.

Surveys:

- Mapping Class Group Dynamics on Surface Group Representations, Goldman, 2006 [Gol06] .
- Dynamics of Aut(Fn) Actions on Group Presentations and Representations, Lubotzky, 2011 [Lub11] .
- *"Dynamics on Character Varieties"*, Canary, 2013 [Can13] . Discussion of both the proper, Anonov side and the work of Minskey, Gelander, etc., for the free groups F_n with values in $SL(2, \mathbb{C})$.

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